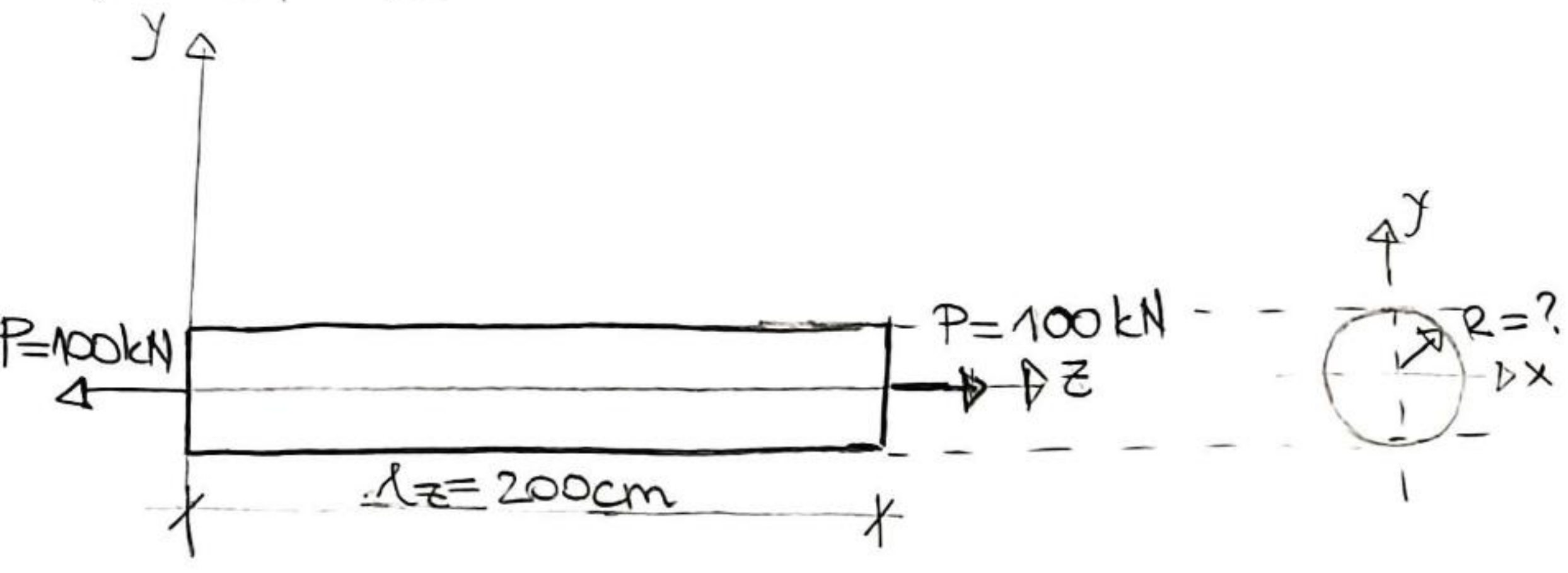


Dimenzionirati stap kružnog poprečnog presjeka ($R = ?$) koji je opterećen aksijalnom silom zatezanja prema seri. $\sigma_{dop} = 16 \text{ kN/cm}^2$. Odredi ti:

- a) dijagram normalnih napona u proizvoljnom poprečnom presjeku štapa
- b) izduženje štapa ($\Delta L = ?$)
- c) tenzor napona i deformacije za proizvoljnu tačku štapa

$\sigma_{dop} = 16 \text{ kN/cm}^2$
 $E = 150 \text{ GPa}$
 $\nu = 0,2 \perp$



a) $\sigma_z = \frac{N}{A} \leq \sigma_{dop}$

$$A_{pot} \geq \frac{N}{\sigma_{dop}} = \frac{100 \text{ kN}}{16 \frac{\text{kN}}{\text{cm}^2}} = 6,25 \text{ cm}^2$$

$$A_{pot} = R^2 \pi, R^2 \geq \frac{A_{pot}}{\pi}, R \geq \sqrt{\frac{A_{pot}}{\pi}} \geq 1,41 \text{ cm}$$

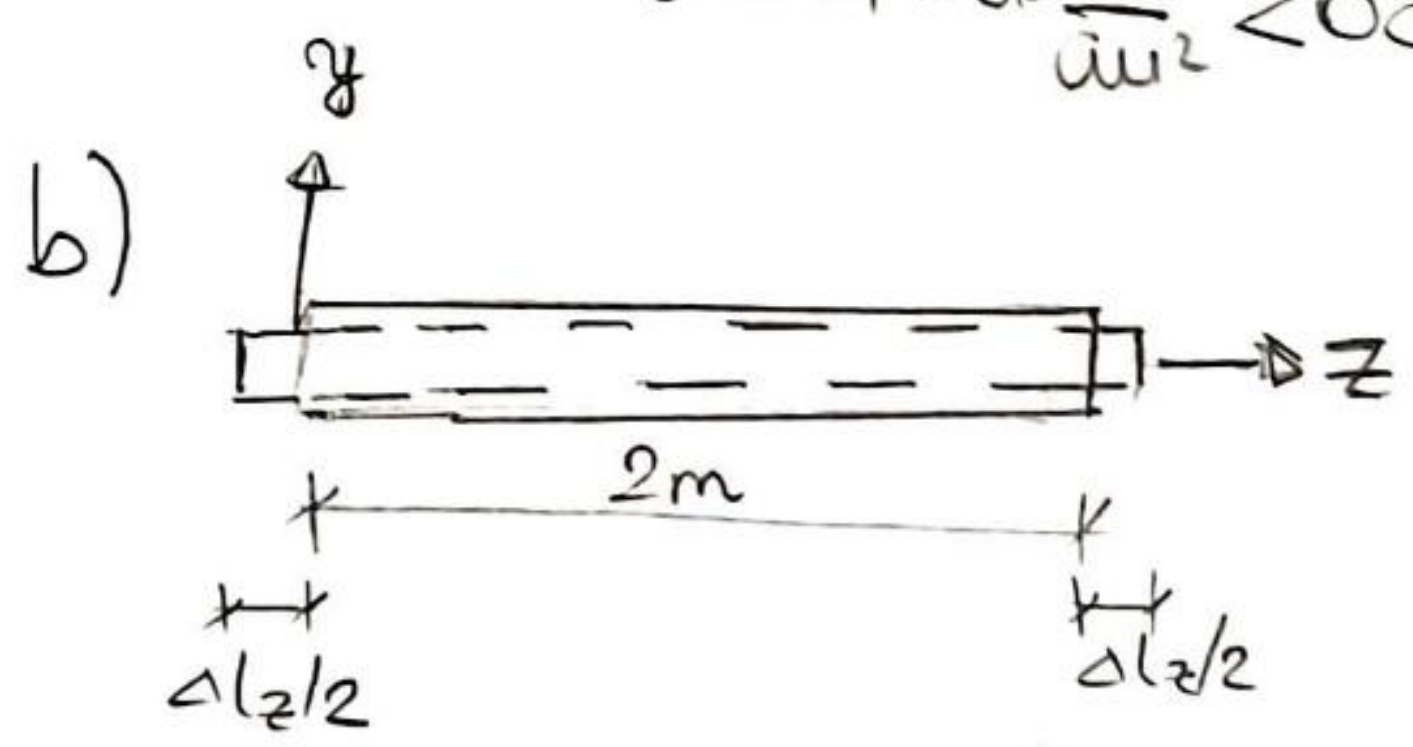
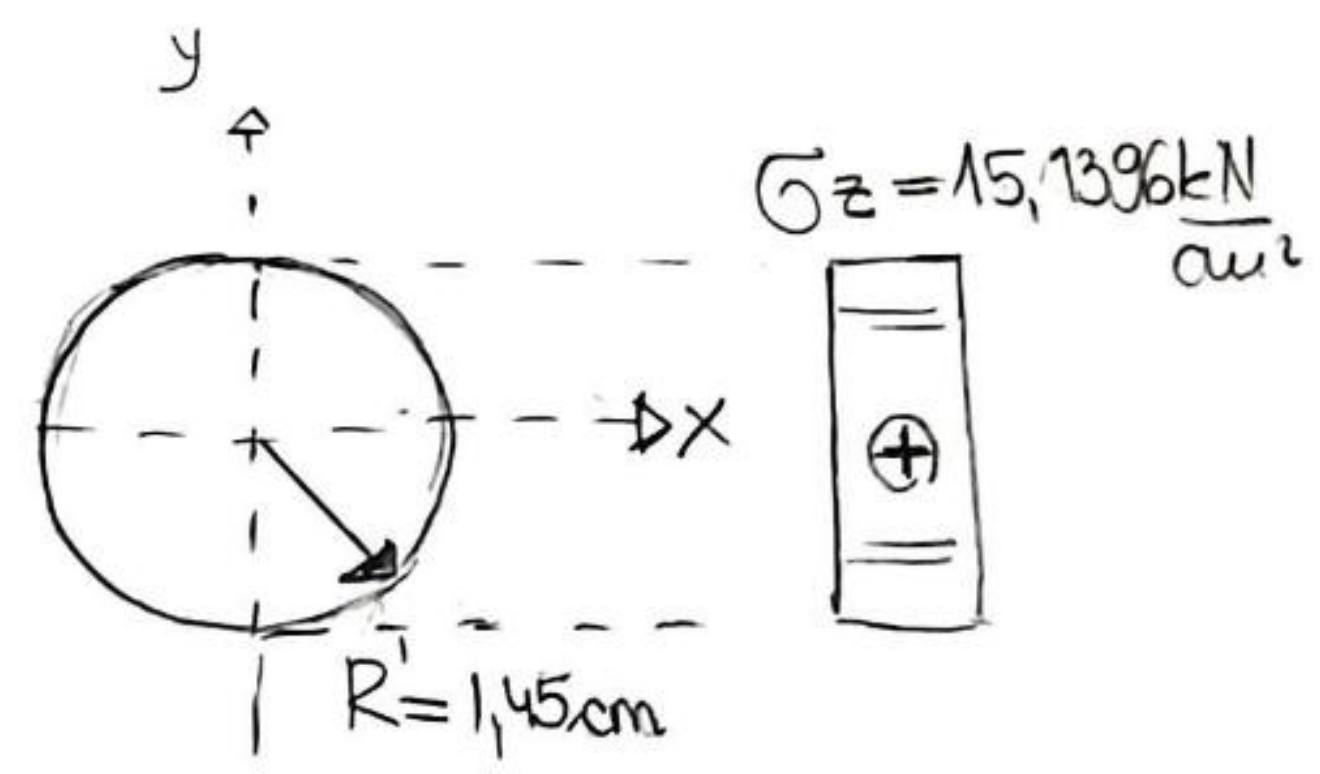
$R = 1,45 \text{ cm}$

$$\sigma = \frac{N}{A}$$

$$A = R^2 \pi = 1,45^2 \pi = 6,6052 \text{ cm}^2$$

$$\sigma = \frac{100 \text{ kN}}{6,6052} = 15,1396 \frac{\text{kN}}{\text{cm}^2}$$

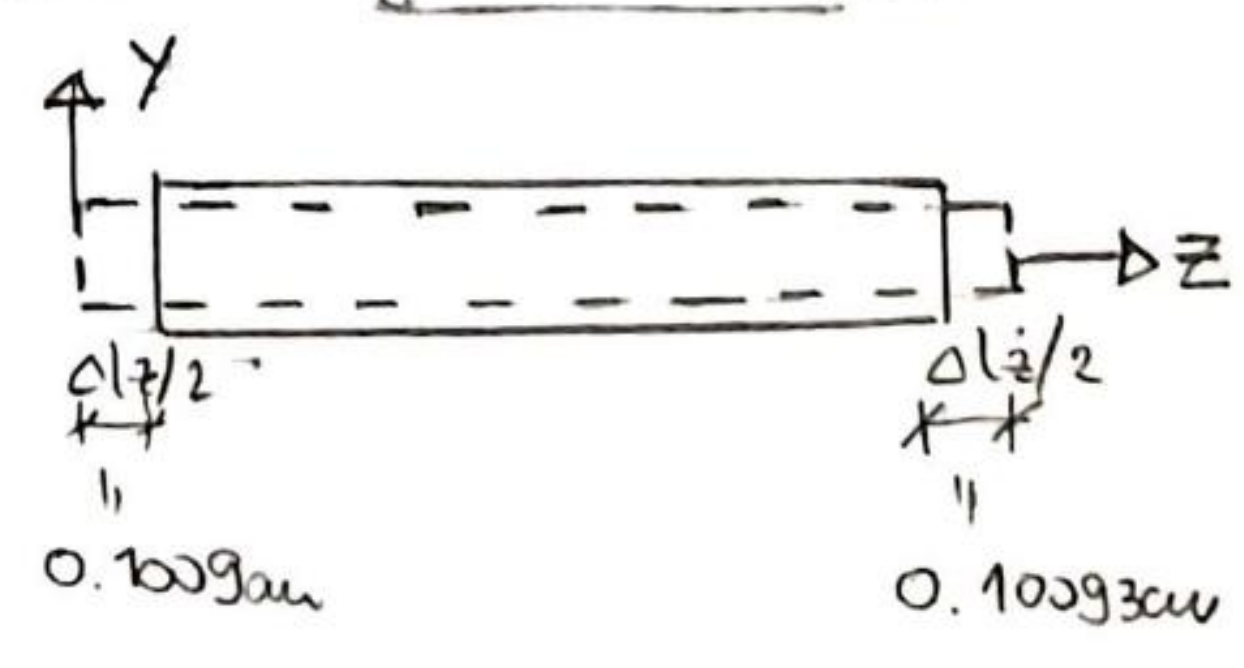
$$\sigma = 15,1396 \frac{\text{kN}}{\text{cm}^2} < \sigma_{dop}$$



$$\epsilon_z = \frac{\sigma_z}{E} = \frac{N}{EA} = \frac{\Delta L_z}{L_z}$$

$$\Delta L_z = \frac{N \cdot L_z}{EA} = \frac{100 \text{ kN} \cdot 200 \text{ cm}}{150 \cdot 10^9 \cdot 6,6052 \text{ cm}^2}$$

$\Delta L_z = 0,20186 \text{ cm}$



$$\sigma = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sigma_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 15.1396 \end{bmatrix} \text{ (kN/cm}^2\text{)} - \text{tenzor napona}$$

$$\varepsilon_x = \varepsilon_y = \frac{\nu}{E} \cdot \sigma_z = -\frac{0,2}{150 \cdot 10^2} \cdot 15.1396 = -2,0186 \cdot 10^{-4}$$

$$\varepsilon_z = \frac{\sigma_z}{E} = \frac{15.1396}{150 \cdot 10^2} = 1,0093 \cdot 10^{-3}$$

$$\mathbb{D} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} = \begin{bmatrix} -0,20186 & 0 & 0 \\ 0 & -0,2018 & 0 \\ 0 & 0 & 1,0093 \end{bmatrix} 10^{-3} - \text{tenzor deformacije}$$

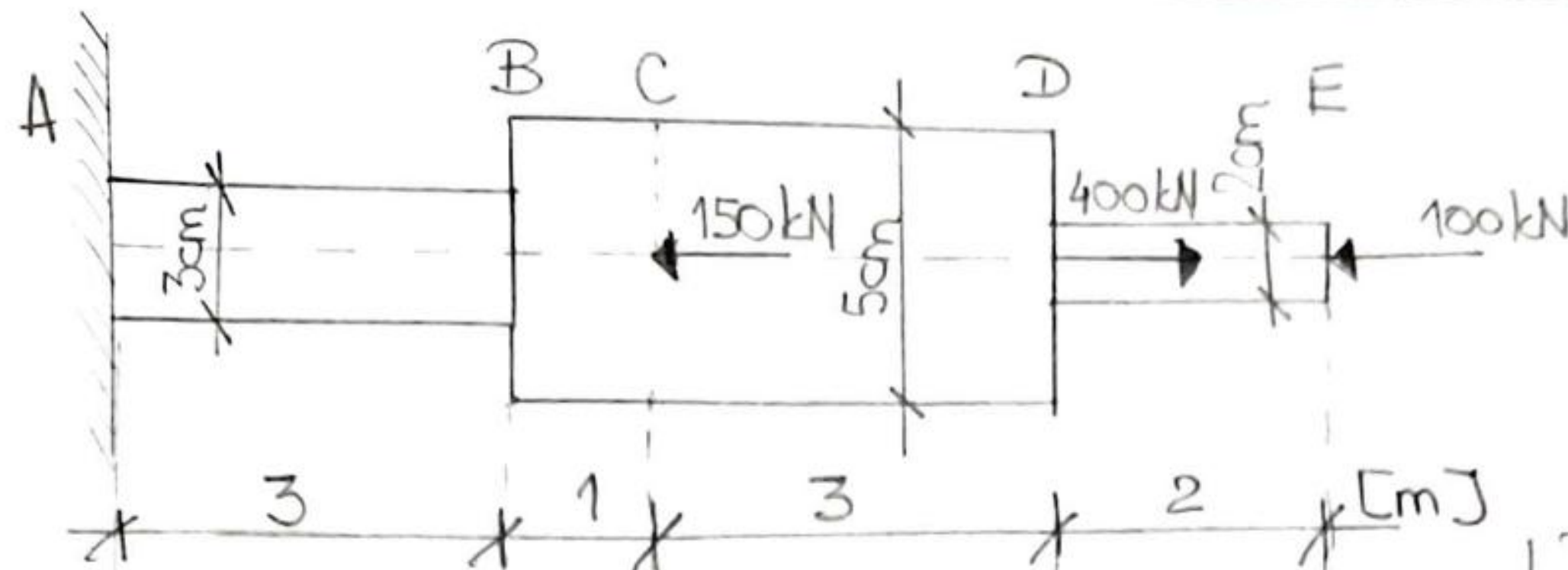
2. Štap skokovitog kvadratnog poprečnog presjeka ($E = 210 \text{ GPa}$, $\Delta t = 1 \cdot 10^{-5} \cdot \frac{1}{c}$), opterećen je kao na slici. Odrediti:

a) dijagram normalnih sila (N) duž nosača

b) dijagram normalnih napona duž nosača i nacrtati dijagram normalnog napona u presjecima E i A

c) dijagram podužnih pomjeranja (naznačiti pomjeranja u tačkama A, B, C, D, E)

d) dijagram podužnih pomjeranja iz slučaja c) pod uslovom da se štap zagrije za 30°C.



a) Uslobo ravnoteže:
 $\sum N = 0, N_A + 150 + 100 - 400 = 0$
 $N_A = 150 \text{ kN}$

b) dio A-B:

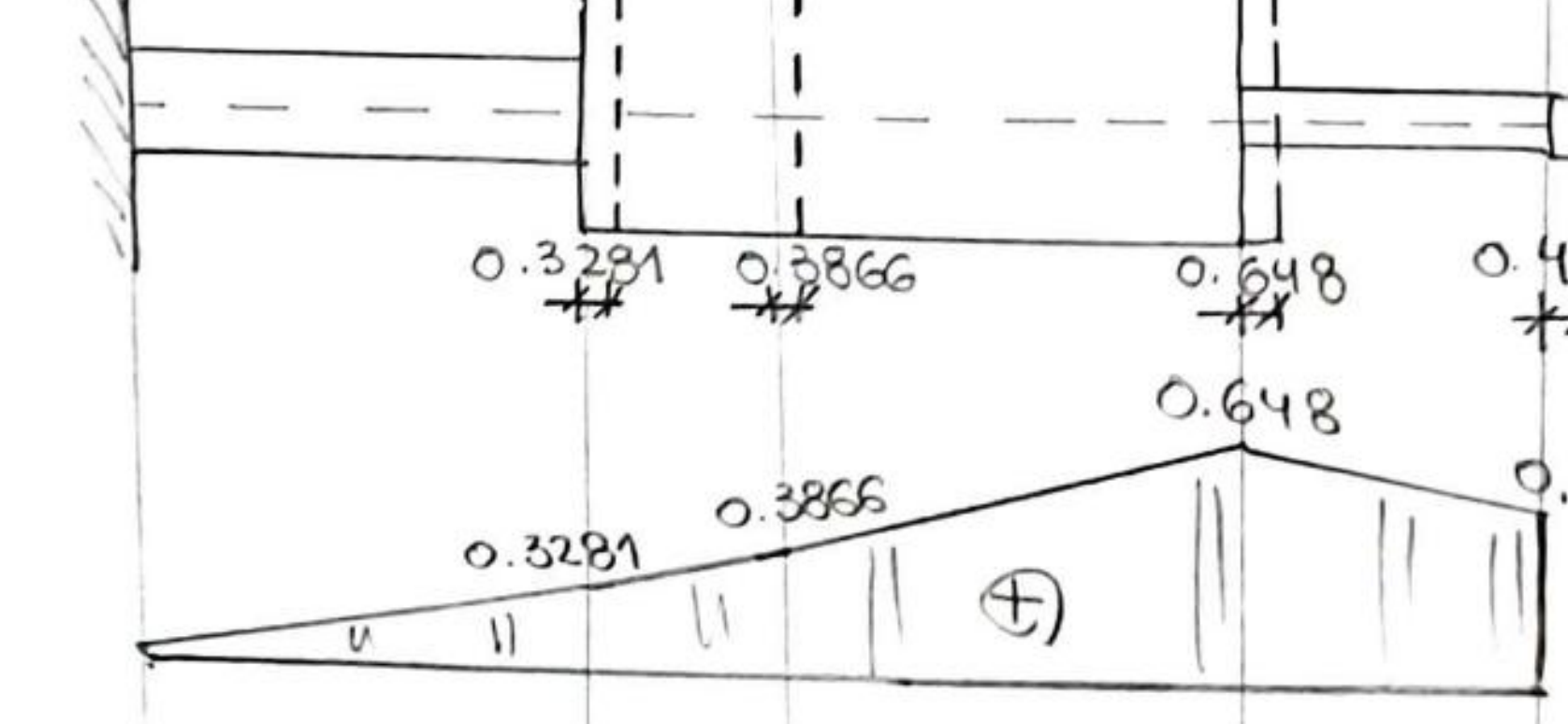
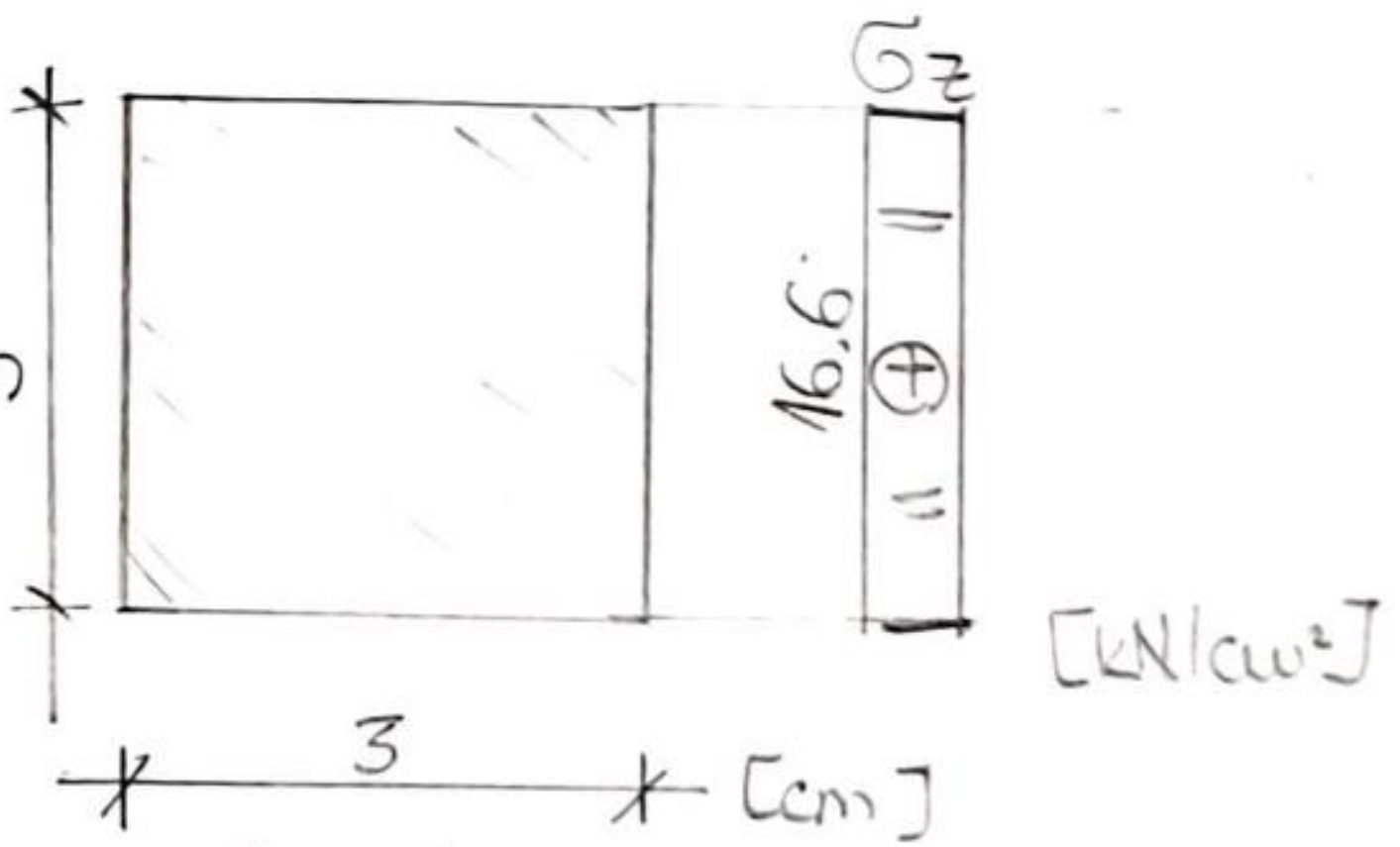
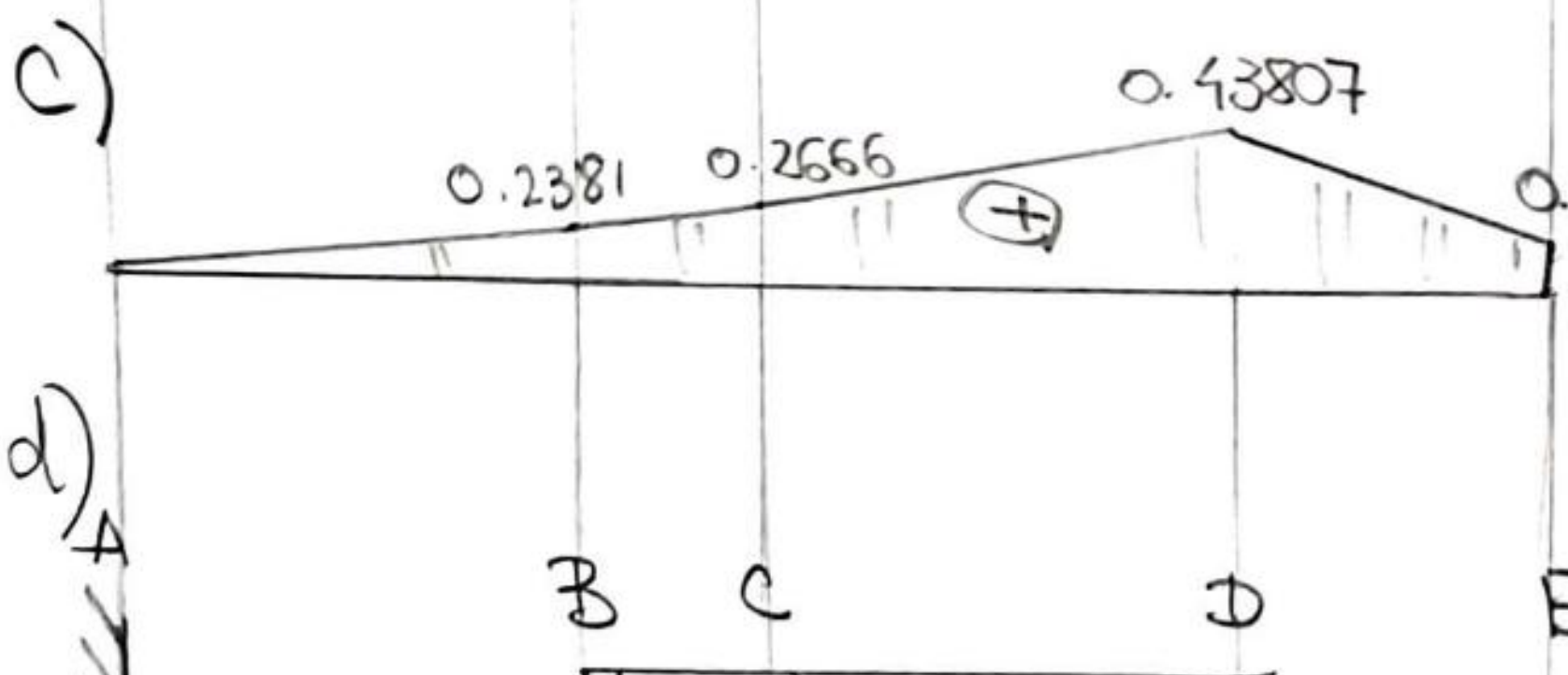
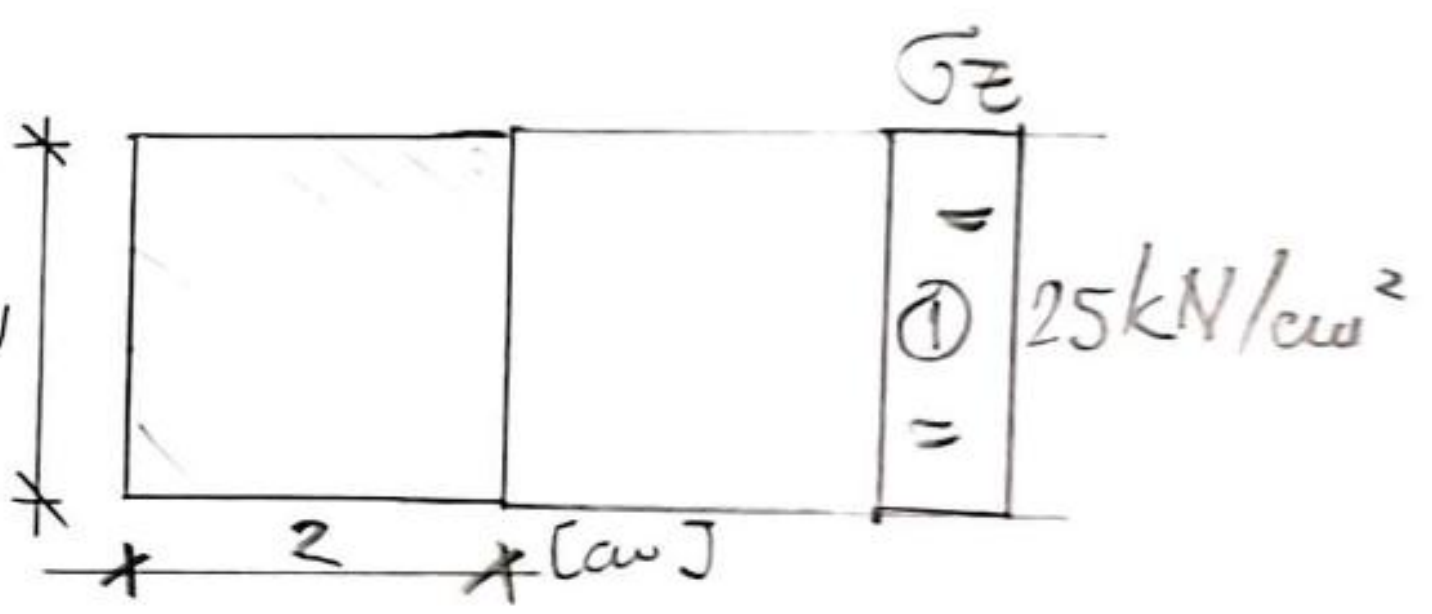
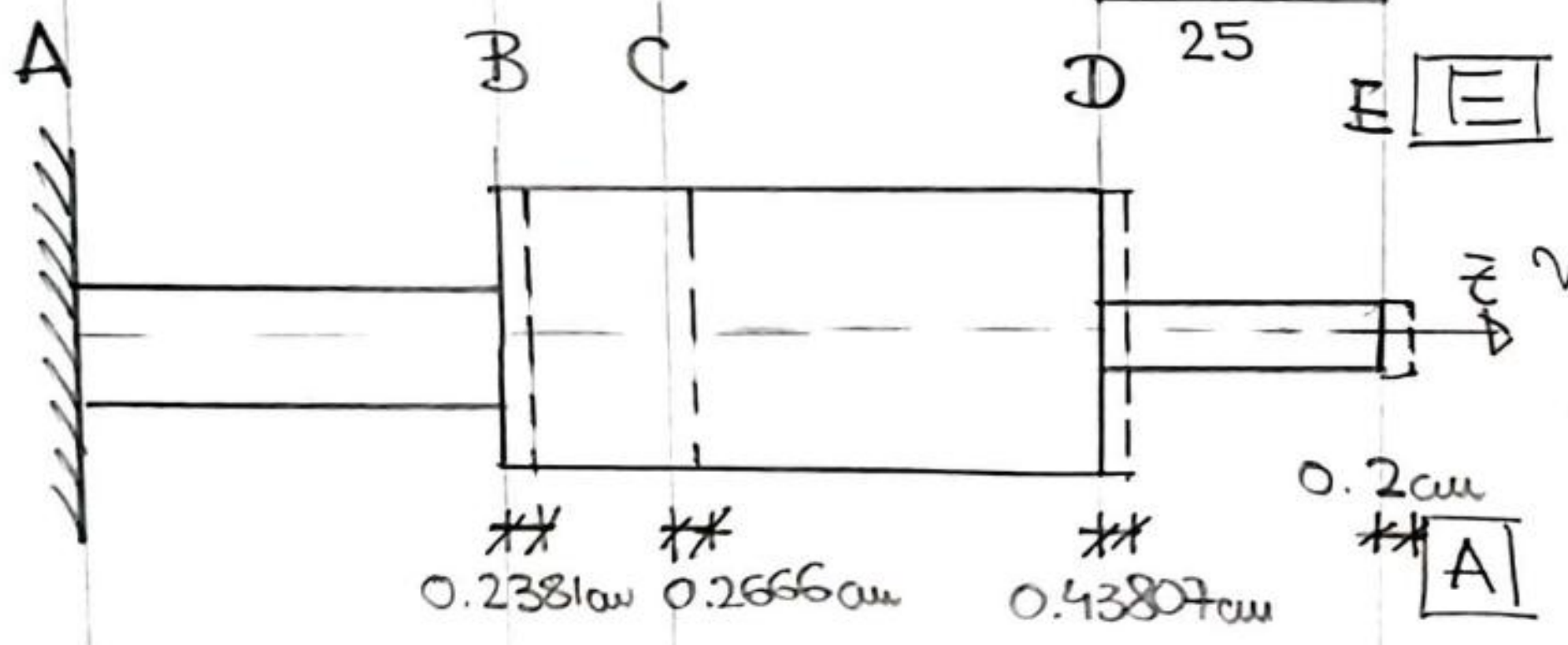
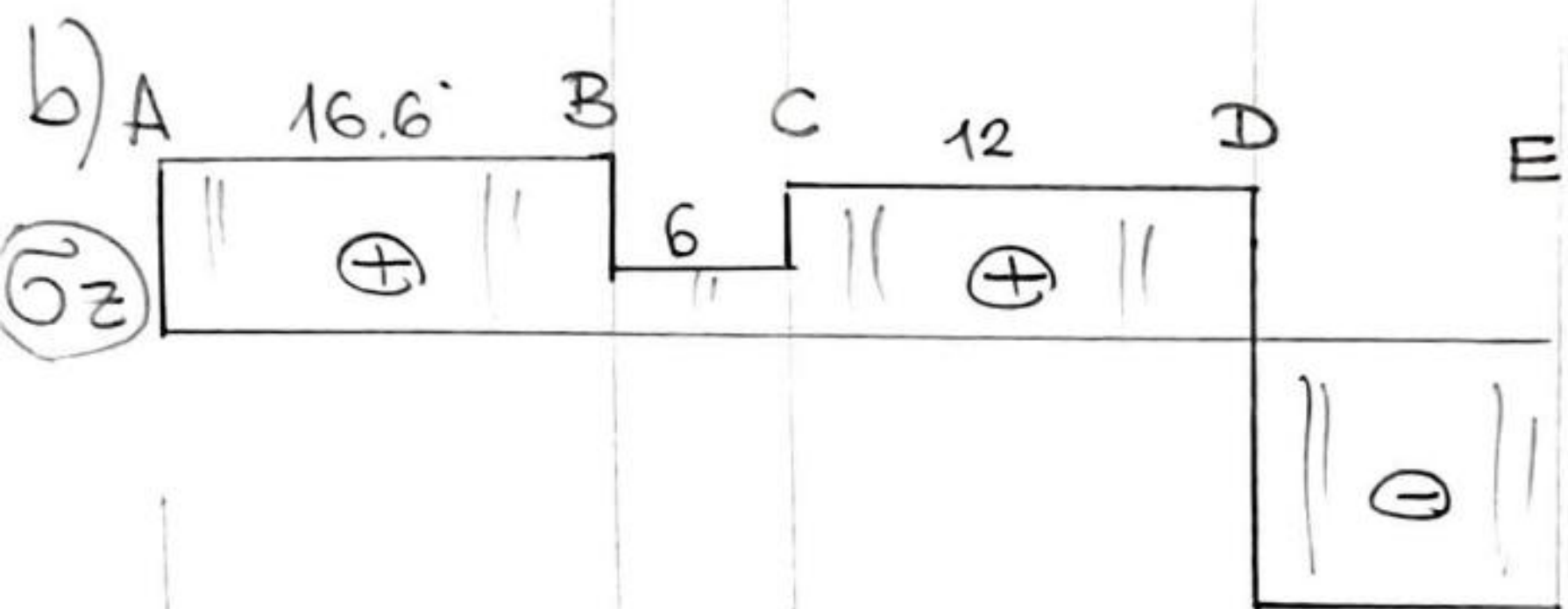
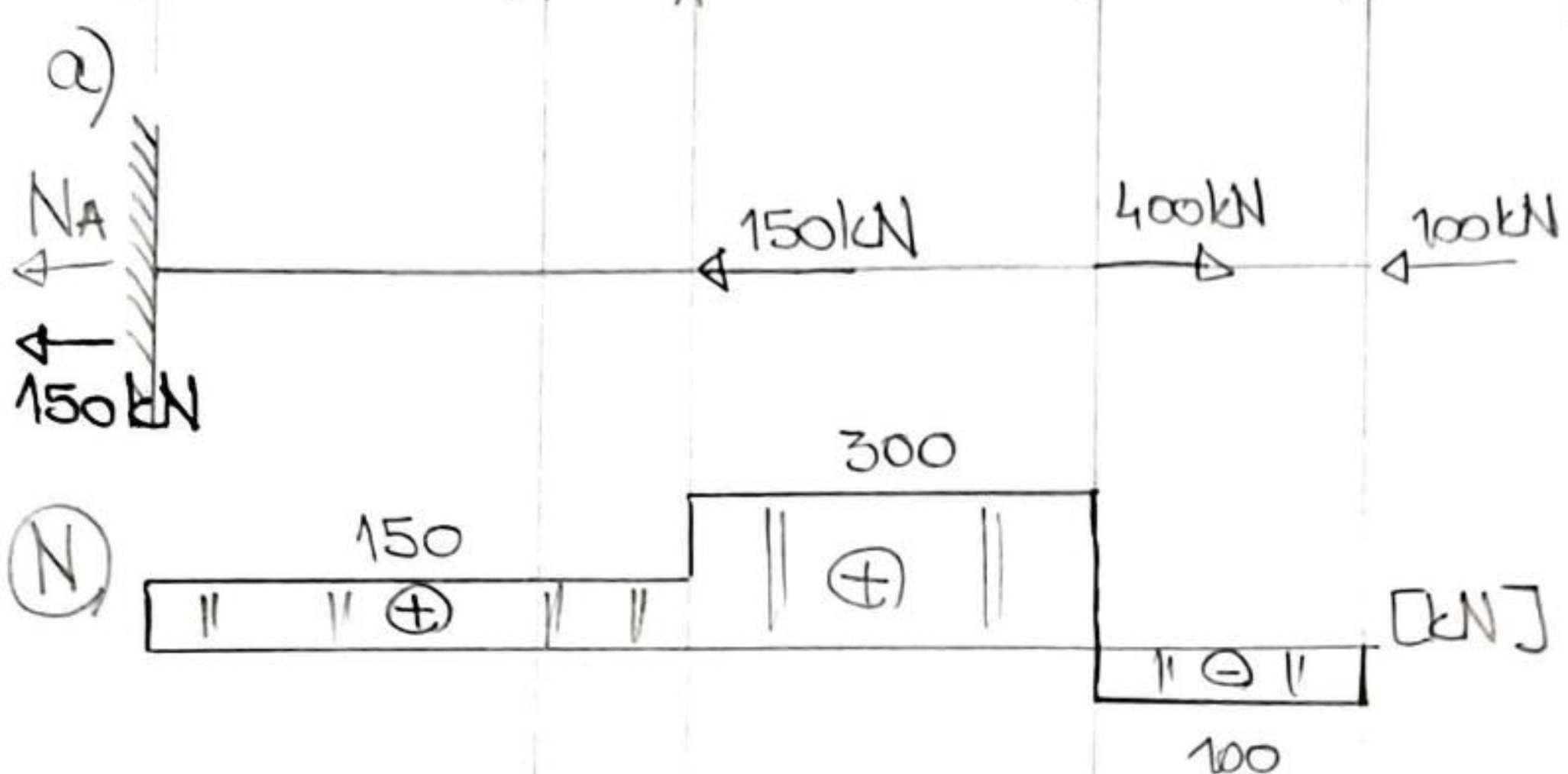
$\sigma_z = \frac{N}{A} = \frac{150}{9} = 16,6 \text{ kN/cm}^2$ (zatezanje)
 $A_{AB} = 3^2 = 9 \text{ cm}^2$

dio B-C:

$\sigma_z = \frac{150}{25} = 6 \text{ kN/cm}^2$ (zatezanje)
 $A_{BC} = A_{CD} = 5 \cdot 5 = 25 \text{ cm}^2$

dio C-D: $\sigma_z = \frac{300}{25} = 12 \text{ kN/cm}^2$ (zatezanje)

[kN/cm²] dio D-E: $\sigma_z = -\frac{100}{4} = -25 \text{ kN/cm}^2$ (pritisanje)
 $A_{DE} = 2 \cdot 2 = 4 \text{ cm}^2$



c) $\Delta l_{AB} = \frac{N_{AB} \cdot l_{AB}}{E \cdot A_{AB}} = \frac{150 \cdot 300 \text{ cm}}{210 \cdot 10^3 \cdot 9} = 0,2381 \text{ cm}$
 $\Delta l_{BC} = \frac{N_{BC} \cdot l_{BC}}{E \cdot A_{BC}} = \frac{150 \cdot 100}{210 \cdot 10^3 \cdot 25} = 0,02857 \text{ cm}$
 $\Delta l_{CD} = \frac{N_{CD} \cdot l_{CD}}{E \cdot A_{CD}} = \frac{300 \cdot 300}{210 \cdot 10^3 \cdot 25} = 0,1714 \text{ cm}$
 $\Delta l_{DE} = \frac{N_{DE} \cdot l_{DE}}{E \cdot A_{DE}} = \frac{-100 \cdot 200}{210 \cdot 10^3 \cdot 4} = -0,2381 \text{ cm}$

$$\Delta L_A = 0$$

$$\Delta L_B = \Delta L_{AB} = 0,2381 \text{ cm}$$

$$\Delta L_C = \Delta L_{AB} + \Delta L_{BC} = 0,2381 + 0,02857 = 0,2666 \text{ cm}$$

$$\Delta L_D = \Delta L_{AB} + \Delta L_{BC} + \Delta L_{CD} = 0,2381 + 0,02857 + 0,1714 = 0,43807 \text{ cm}$$

$$\Delta L_E = \Delta L_{AB} + \Delta L_{BC} + \Delta L_{CD} + \Delta L_{DE} = 0,2381 + 0,02857 + 0,1714 - 0,2381 = 0,2 \text{ cm}$$

$$d) \Delta L_B = 0,2381 + \alpha_t \cdot \Delta t \cdot L_B = 0,2381 + 10^{-5} \cdot 30 \cdot 300 = 0,3281 \text{ cm}$$

$$\Delta L_C = 0,2666 + 10^{-5} \cdot 30 \cdot 400 = 0,3866 \text{ cm}$$

$$\Delta L_D = 0,43807 + 10^{-5} \cdot 30 \cdot 700 = 0,64807 \text{ cm}$$

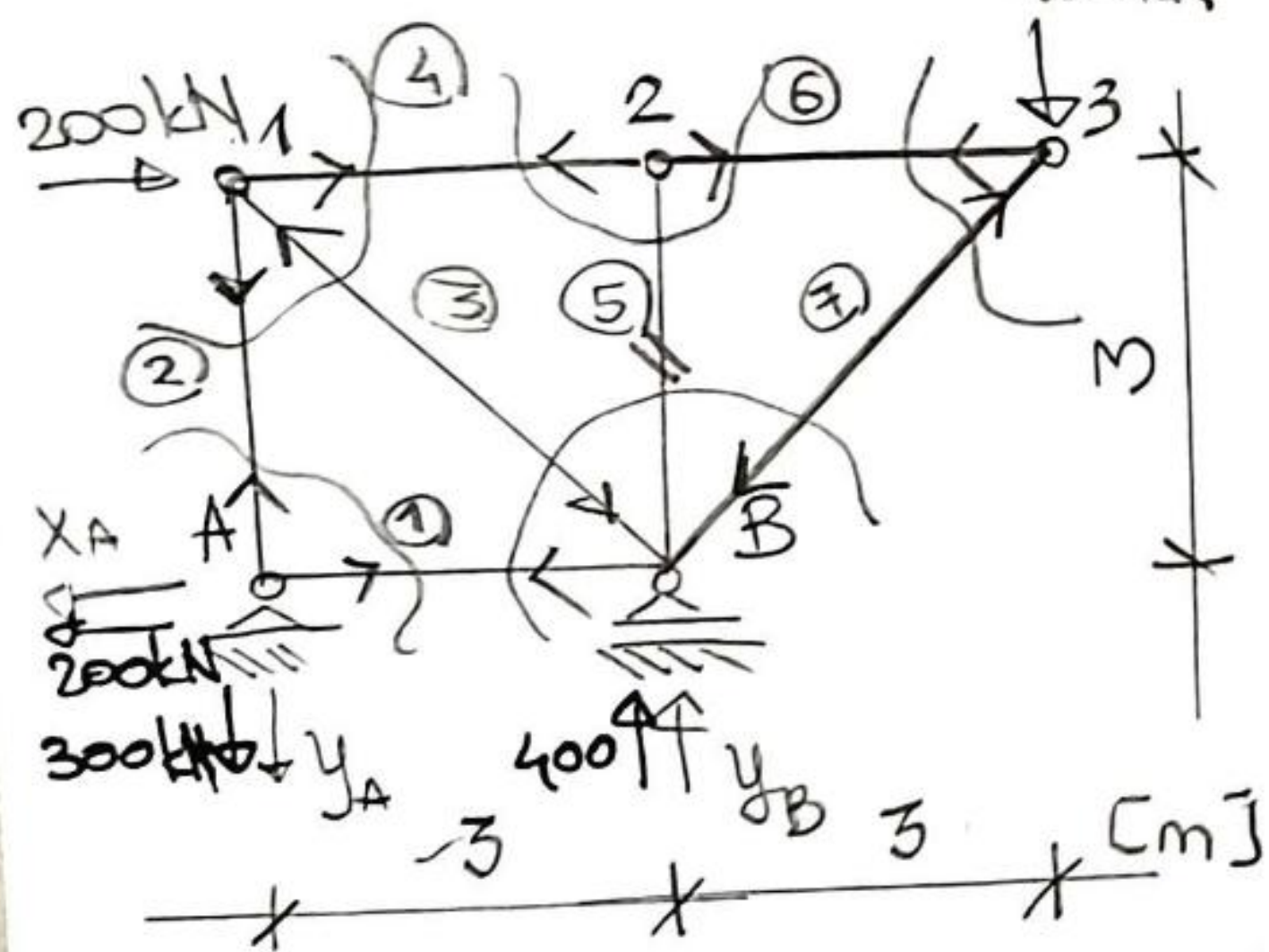
$$\Delta L_E = 0,2 + 10^{-5} \cdot 30 \cdot 900 = 0,47 \text{ cm}$$

3. Za rešetkasti nosač odrediti:

a) Vrijednosti aksijalnih sila metodom isijecanja čvorova

b) Dimenzionirati nosač, ako je $\sigma_{\text{dop}} = 10 \text{ kN/cm}^2$

c) Izračunati maksimalno izduženje i maksimalno skraćenje štapova ako je $E = 200 \text{ GPa}$.



a) Uslovi ravnoteže:

$$\sum X = 0 \quad (1) \quad \sum M_B = 0$$

$$X_A - 200 = 0$$

$$200 \cdot 3 + 100 \cdot 3 - Y_A \cdot 3 = 0 \quad (2)$$

$$X_A = 200 \text{ kN}$$

$$Y_A = 300 \text{ kN}$$

$$\sum Y = 0, \quad -300 + Y_B - 100 = 0, \quad Y_B = 400 \text{ kN}$$

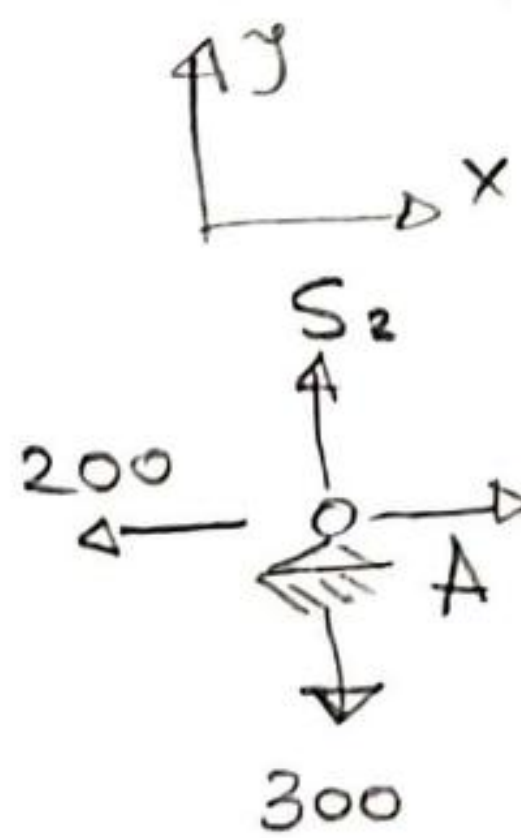
Kontrola:

$$\sum M_A = 0 \quad (f)$$

$$400 \cdot 3 - 100 \cdot 6 - 200 \cdot 3 = 0$$

$$0 = 0 \quad (T)$$

čvor A:



$$\sum X = 0$$

$$S_1 - 200 = 0$$

$$S_1 = 200 \text{ kN} \quad (\text{zat.})$$

$$\sum Y = 0$$

$$S_2 - 300 = 0$$

$$S_2 = 300 \text{ kN} \quad (\text{zatezanje})$$

čvor 1:



$$200$$

$$S_4$$

$$300$$

$$\sum Y = 0$$

$$-300 - S_3 \frac{\sqrt{2}}{2} = 0$$

$$S_3 = -424,264 \text{ kN} \quad (\text{pritisanje})$$

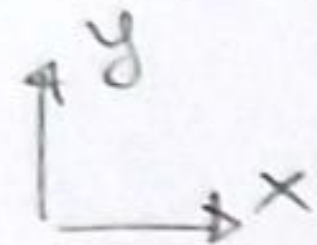
$$\sum X = 0$$

$$200 + S_4 + S_3 \frac{\sqrt{2}}{2} = 0$$

$$S_4 = -200 - S_3 \frac{\sqrt{2}}{2}$$

$$S_4 = 100 \text{ kN} \text{ (zatezawa)}$$

čvor 2:



$$\sum X = 0$$

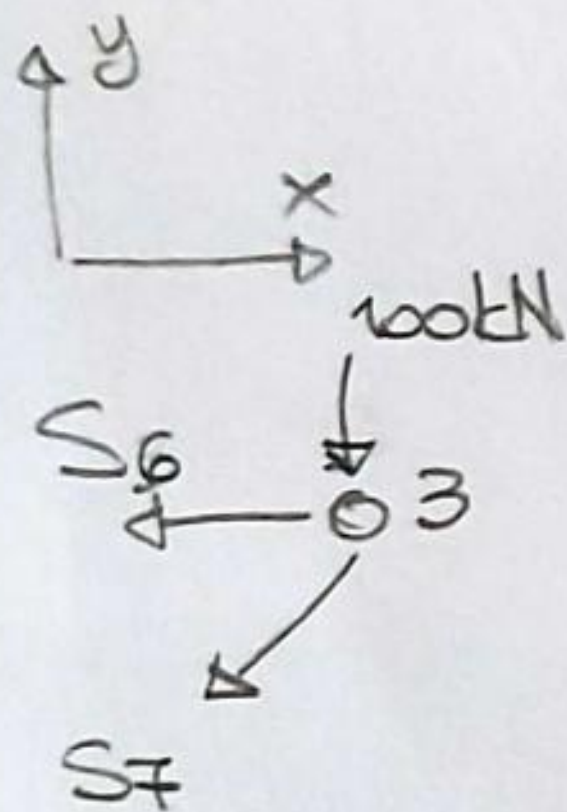
$$-S_4 + S_6 = 0$$

$$S_6 = S_4 = +100 \text{ kN} \text{ (zatezawa)}$$

$$\sum Y = 0$$

$$S_5 = 0$$

čvor 3:



$$\sum X = 0$$

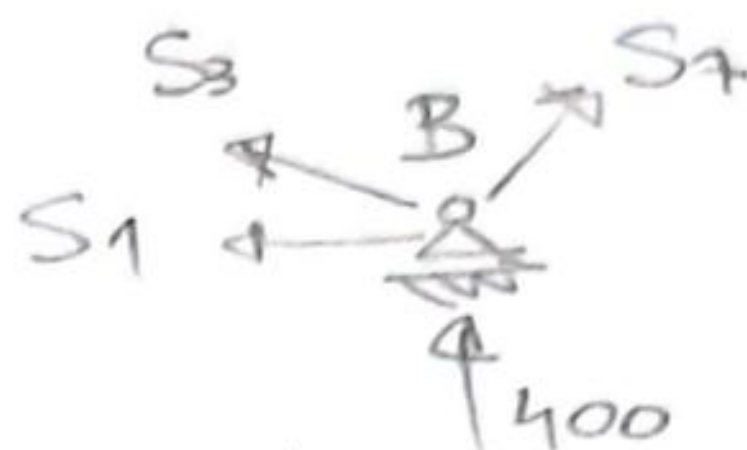
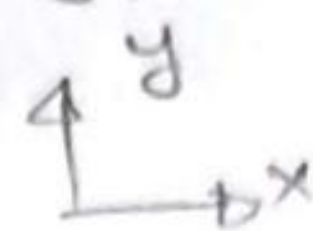
$$-S_6 - S_7 \frac{\sqrt{2}}{2} = 0$$

$$S_7 = -S_6 \cdot \frac{2}{\sqrt{2}}$$

$$S_7 = -141,42 \text{ kN} \text{ (pritiska)}$$

provjera:

čvor B:



$$\sum X = 0$$

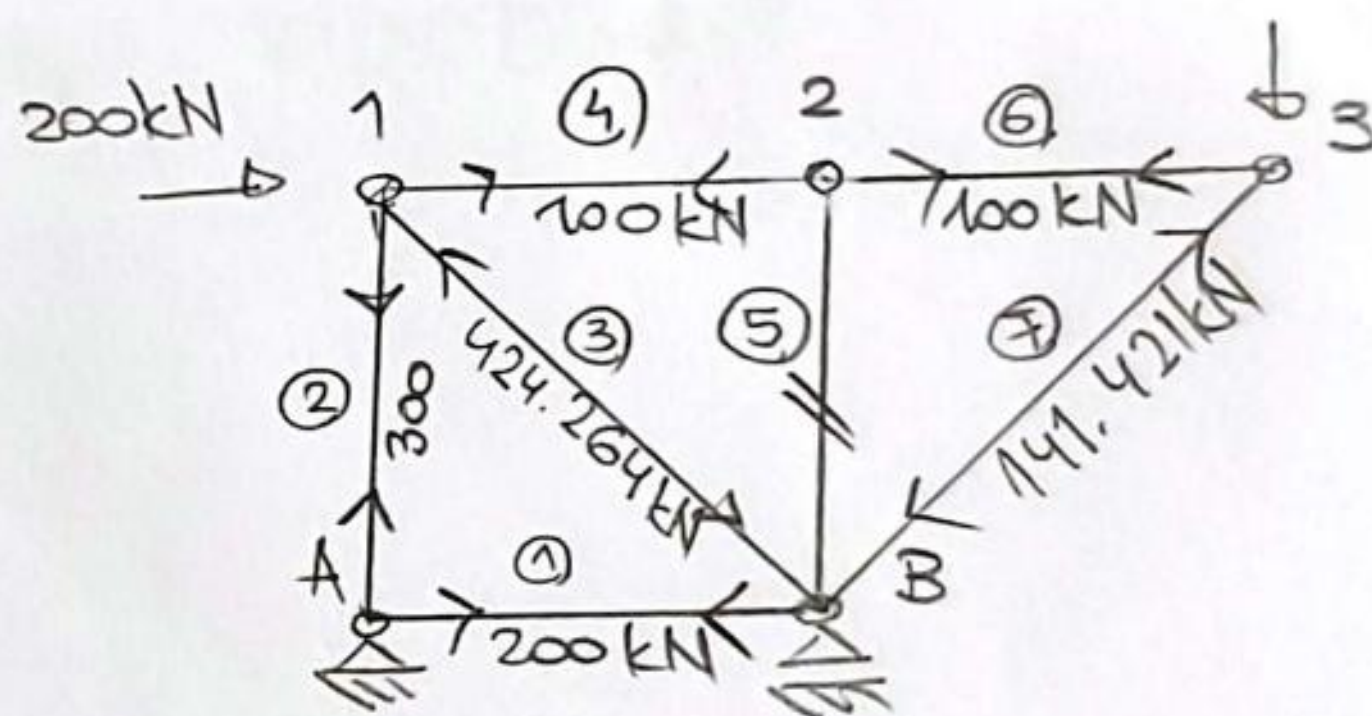
$$-S_3 \frac{\sqrt{2}}{2} + S_7 \frac{\sqrt{2}}{2} - S_1 = 0$$

$$300 - 100 - 200 = 0 \text{ (T) u}$$

$$\sum Y = 0$$

$$400 + S_3 \frac{\sqrt{2}}{2} + S_7 \frac{\sqrt{2}}{2} = 0$$

$$400 - 300 - 100 = 0 \text{ (T) u}$$



b) $\sigma_{dop} = 10 \text{ kN/cm}^2$

$$\sigma_z = \frac{|N|}{A} \leq \sigma_{dop} \Rightarrow A_{pot} \geq \frac{|N|}{\sigma_{dop}}$$

$$A_{pot} \geq \frac{|-424,264|}{10} = 42,426 \text{ cm}^2, \text{ usvaja se } \boxed{A = 42,5 \text{ cm}^2}$$

c) maks. izduženje:

$$S_2 = 300 \text{ kN} \text{ (max. sila zatezawa)}$$

$$\Delta l_2 = \frac{S_2 \cdot l_2}{E \cdot A_2} = \frac{300 \cdot 300}{200 \cdot 10^4 \cdot 42,5} = 0,10588 \text{ cm}$$

maks. skraćewe:

$$S_3 = -424,264 \text{ kN} \text{ (max sila pritiska)}$$

$$\Delta l_3 = \frac{-424,264 \cdot 4,242 \cdot 10^2}{200 \cdot 10^4 \cdot 42,5} = -0,21173 \text{ cm}$$

$$l_3 = 3\sqrt{2} = 4,242 \text{ m}$$